HEAT TRANSFER DURING CONDENSATION OF VAPORS ON A JET

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Relations determining heat transfer during condensation of vapor on a jet with consideration of the thermal resistance to mass transfer of vapor toward the jet were obtained and the results of calculations are presented.

Jet condensers and water heaters are well-known and have been used for a long time in engineering, but the presently accumulated experimental and theoretical investigations on vapor condensation on a jet are rather scanty and pertain mainly to condensation of water vapor.

The relation obtained by S. S. Kutateladze [1] for determining the magnitude of relative underheating is well-known:

$$\vartheta_{av} = \frac{T_v - T_{av}}{T_v - T_0} = 4 \sum_{i=1}^{\infty} \exp\left[-\alpha_i^2 f(X)\right] \frac{1}{\alpha_i^2}. \tag{1}$$

However, this expression was obtained on the assumption that the surface temperature of the jet is equal to the vapor temperature, i.e., that the resistance to mass transfer of the vapor toward the jet surface can be neglected.

For working bodies with low thermal conductivity and for small specific volumes of vapor this assumption is valid with sufficient accuracy and the calculations made for water vapor agree satisfactorily with the experimental results. For working bodies with a high thermal conductivity or for large specific volumes of vapor the thermal resistance to mass transfer becomes comparable with resistance to heat transfer into the inner layers of the jet and under certain conditions is decisive.

In connection with the decrease of the cross section on approach to the jet surface the vapor expands, its temperature drops, and the velocity increases. However, like the process of expansion in a convergent nozzle the velocity of the vapor cannot increase above the velocity of sound and the flow rate of vapor cannot be greater than the critical. As a consequence of this, at a surface temperature of the jet equal to or greater than the critical a critical vapor flow is established in the immediate vicinity of the surface, the vapor velocity is equal to the velocity of sound, and the vapor temperature and its flow rate are equal respectively to T_{cr} and $j_{cr} = f(T_v)$. For a jet surface temperature greater than T_{cr} a subcritical vapor flow is established near the jet surface and in this case the vapor flow rate is a function of the stagnation temperature of the vapor and the jet surface temperature:

$$j = f(T_v \;\; T_{sur}).$$

Thus in a general form, over the length of the jet there is:

I. A section with $T_{sur} \le T_{cr}$ and constant specific vapor flow rate $j = j_{cr} = j(T_v)$.

II. A section with $T_{sur} > T_{cr}$ and specific vapor flow rate $j = f(T_v,\ T_{sur})$ for $j = j_{cr}\,\varphi\ (T_v,\ T_{sur})$.

The temperature at any point of the jet is determined by the equation of thermal conductivity

$$\frac{w_0 C\gamma}{\lambda + \lambda_t} \cdot \frac{\partial T}{\partial X} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial T}{\partial r}. \tag{2}$$

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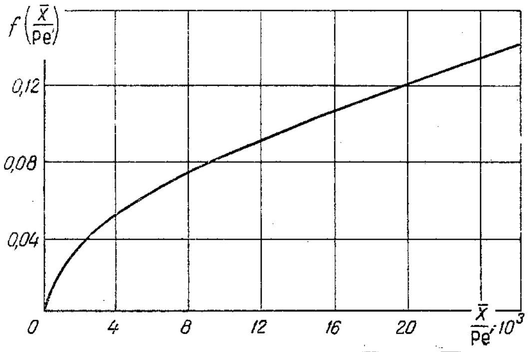

Fig. 1. Function $f(\overline{X}/Pe') = (\overline{T}_{sur} - \overline{T}_0)/K$ for the section with a critical vapor flow near the jet surface.

Introducing the dimensionless coordinates and temperature $\overline{r} = r/r_0$, $\overline{X} = X/r_0$, and $\overline{T} = T/T_v$, we obtain

$$\frac{r_0 w_0 C\gamma}{\lambda + \lambda_t} \cdot \frac{\partial \overline{T}}{\partial \overline{X}} = \frac{\partial^2 \overline{T}}{\partial \overline{r}^2} + \frac{1}{\overline{r}} \cdot \frac{\partial \overline{T}}{\partial \overline{r}}. \tag{3}$$

For section I the boundary conditions are $\overline{T} = \overline{T}_0$ for $\overline{X} = 0$ and $q = q_{cr}$ for $\overline{r} = 1$, where q is the specific quantity of heat liberated during condensation of the vapor on the jet surface.

Solving (3) for these conditions and assuming like in [1] λ and λ_t to be independent of $\overline{X}$ and $\overline{r}$, we find

$$\overline{T} = \overline{T}_0 + \frac{2q_{cr}}{T_v w_0 C\gamma} \overline{X} + \frac{q_{cr} r_0}{T_v (\lambda + \lambda_t)} \left\{ \frac{1}{2} \overline{r}^2 - 0.25 - 2 \sum_{n=1}^{\infty} \exp\left[-\beta_n^2 \frac{\lambda + \lambda_t}{r_0 w_0 C\gamma} \overline{X} \right] \frac{J_0(\beta_n \overline{r})}{\beta_n^2 J_0(\beta_n)} \right\},$$

where $J_0(\beta_n \overline{r})$ is a zero-order Bessel function and the β_n are positive roots of the equation $J_1(\beta_n) = 0$.

Introducing the notations $K = q_{cr} d_0 / T_v(\lambda + \lambda_t)$ and $Pe' = Pe/(1 + \lambda_t/\lambda)$, after elementary transformations we obtain the relation for determining the temperature at any point of the jet in section I in the form

$$\overline{T} = \overline{T}_0 + 2K \frac{\overline{X}}{Pe'} + \frac{1}{2} K \left\{ \frac{1}{2} \overline{r}^2 - 0.25 - 2 \sum_{n=1}^{\infty} \exp\left[-2\beta_n^2 \frac{\overline{X}}{Pe'} \right] \frac{J_0(\beta_n \overline{r})}{\beta_n^2 J_0(\beta_n)} \right\}. \tag{4}$$

The jet surface temperature $(\overline{r} = 1)$ is

$$\overline{T}_{sur} = \overline{T}_0 + 2K \frac{\overline{X}}{Pe'} + \frac{1}{2} K \left\{ 0.25 - \sum_{n=1}^{\infty} \exp\left[-2\beta_n^2 \frac{\overline{X}}{Pe'} \right] \frac{1}{\beta_n^2} \right\} = \overline{T}_0 + Kf\left(\frac{\overline{X}}{Pe'} \right). \tag{5}$$

The values of $f(\overline{X}/Pe')$ are presented in Fig. 1. The boundary of section I $\overline{X}_{cr}$ can be determined from the condition $\overline{T}_{sur} = \overline{T}_{cr}$ for given K and $\overline{T}_0$.

The average temperature in any cross section

$$\overline{T}_{cr} = 2 \int_0^1 \overline{T}\, \overline{r}\, d\overline{r} = \overline{T}_0 + 2K \frac{\overline{X}}{Pe'}.$$

This same expression can be obtained directly from the heat balance on the jet. The average relative underheating of the jet in section I is

$$\vartheta_{av}^{I} = 1 - 2 \frac{K}{1 - \overline{T}_0} \cdot \frac{\overline{X}}{Pe'}. \tag{6}$$

For section II the boundary condition is

$$(\lambda + \lambda_t) \frac{\partial T}{\partial r} \bigg|_{r=r_0} - q = 0, \text{ where } q = q_{cr} \frac{j}{j_{cr}}. \tag{7}$$

In the general form q is a nonlinear function of the vapor temperature and jet surface temperature. The solution of the equation of thermal conductivity with a nonlinear boundary condition is extremely difficult. Only several exact solutions of problems of thermal conductivity with nonlinear heat transfer are known. In particular, the case of a semibounded solid with heat transfer by radiation is considered in [2]. At the same time, for small temperature drops the function q can be approximated with a degree of accuracy sufficient for determining underheating by the linear dependence

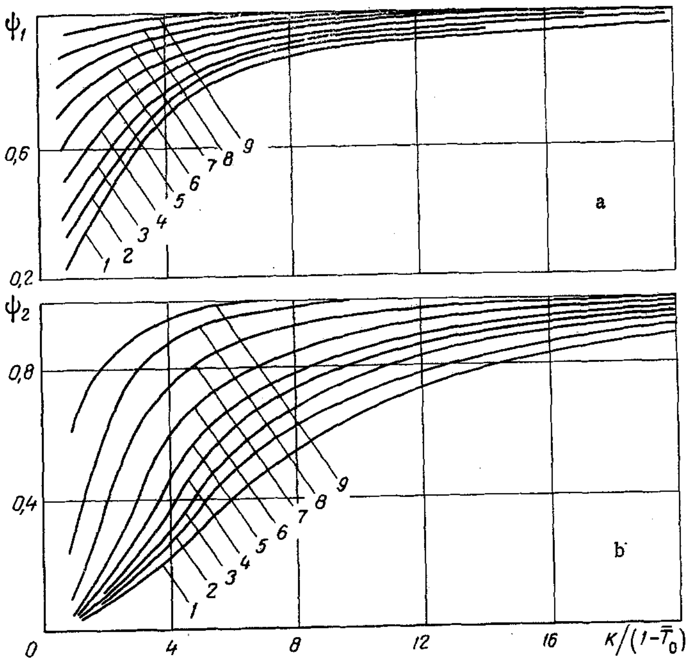

Fig. 2. Coefficient ψ_i for the first term of the series (a) and for the second term of the series (b): 1) $(T_V-T_{cr})/(T_V-T_0)=0.1$; 2) 0.2; 3) 0.3; 4) 0.4; 5) 0.5; 6) 0.6; 7) 0.7; 8) 0.8; 9) 0.9.

$$q = q_{cr}\frac{T_V - T_{sur}}{T_V - T_{cr}}.$$

Introducing the notation $\vartheta = (T_V - T)/(T_V - T_0)$, we obtain the equation of thermal conductivity for section II in the from

$$\frac{Pe'}{2}\cdot\frac{\partial\vartheta}{\partial(\overline{X}-\overline{X}_{cr})} = \frac{\partial^2\vartheta}{\partial\overline{r}^2} + \frac{1}{\overline{r}}\cdot\frac{\partial\vartheta}{\partial\overline{r}}. \tag{8}$$

The appropriately transformed boundary condition is

$$\left.\frac{\partial\vartheta}{\partial\overline{r}}\right|_{\overline{r}=1} + \frac{K}{2(1-\overline{T}_{cr})}\left.\vartheta\right|_{\overline{r}=1} = 0. \tag{9}$$

We will seek the solution in the form $\vartheta = \exp\,[-2\alpha^2\,(\overline{X}-X_{cr})/Pe']\,\varphi(\overline{r})$.

Substituting this expression into the initial one, we obtain the equation for determining $\varphi(\overline{r})$:

$$\alpha^2\varphi(\overline{r}) + \frac{\partial^2\varphi(\overline{r})}{\partial\overline{r}^2} + \frac{1}{\overline{r}}\cdot\frac{\partial\varphi(\overline{r})}{\partial\overline{r}} = 0,$$

which represents a zero-order Bessel equation. Since ϑ is a finite quantity when $\overline{r}=0$, its solution of the second kind (Neuman function) is discarded and the solution is obtained in the form of a series

$$\vartheta = \sum_{i=1}^{\infty}\exp\left[-2\alpha_i^2\frac{\overline{X}-\overline{X}_{cr}}{[Pe'}\right]J_0(\alpha_i\overline{r})\,A_i. \tag{10}$$

Substituting (10) into (9), we obtain the expression for determining the coefficients α_i:

$$\alpha_i J_1(\alpha_i) - \frac{K}{2(1-\overline{T}_{cr})}J_0(\alpha_i) = 0. \tag{11}$$

The numerical values of α_i, just as of β_n, are encountered often in the literature. For example, the values of these coefficients for six members of the series are presented in [2].

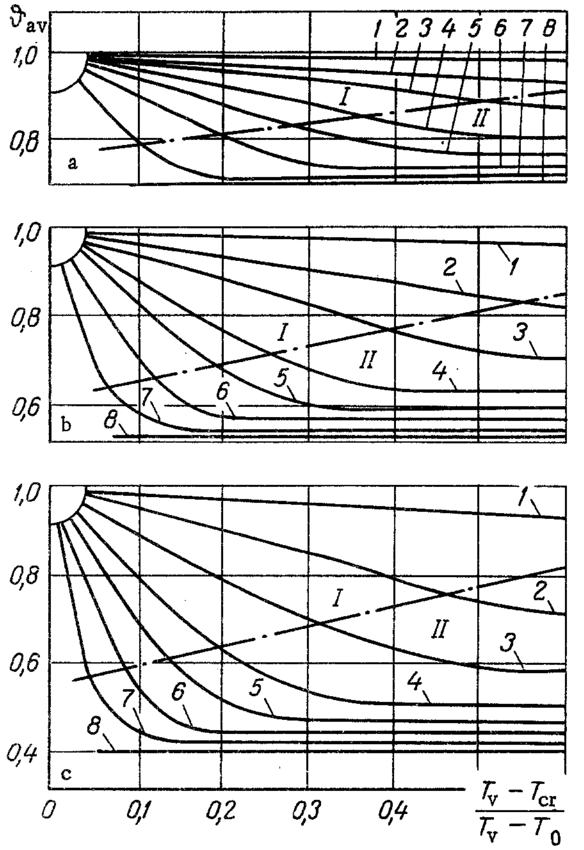

Fig. 3. Average relative underheating for: a) $\overline{X}/Pe' = 1 \times 10^{-2}$; b) $\overline{X}/Pe' = 3 \times 10^{-2}$; c) $\overline{X}/Pe' = 5 \times 10^{-2}$; 1) $K/(1-\overline{T}_{cr}) = 1$; 2) 5; 3) 10; 4) 20; 5) 30; 6) 50; 7) 100; 8) ∞; I) critical vapor flow over entire length of jet; II) critical and subcritical vapor flow over length of jet.

The boundary conditions for section II is the relation obtained from (4) and (10) for $\overline{X} = \overline{X}_{cr}$

$$\vartheta_0^{II} = 1 - \frac{K}{1-\overline{T}_0}\left\{2\frac{\overline{X}_{cr}}{Pe'} + \frac{1}{4}\overline{r}^2 - 0{,}125 - \sum_{n=1}^{\infty}\exp\left[-2\beta_n^2\frac{\overline{X}_{cr}}{Pe'}\right]\frac{J_0(\beta_n\overline{r})}{\beta_n^2 J_0(\beta_n)}\right\} = \sum_{i=1}^{\infty} J_0(\alpha_i\overline{r})A_i.$$

From here we determine

$$A_i = 2\alpha_i^2\frac{\int_0^1 \vartheta_0^{II}\,\overline{r}J_0(\alpha_i\overline{r})\,d\overline{r}}{\left\{\left[\frac{K}{2(1-\overline{T}_{cr})}\right]^2 + \alpha_i^2\right\}[J_0(\alpha_i)]^2}.$$

After integration and appropriate transformation we obtain the expression for determining the temperature at any point of the jet in section II:

$$\vartheta = 4\frac{1-\overline{T}_{cr}}{K}\sum_{i=1}^{\infty}\exp\left[-2\alpha_i^2\frac{\overline{X}-\overline{X}_{cr}}{Pe'}\right] \times \frac{\psi_i\frac{J_0(\alpha_i\overline{r})}{J_0(\alpha_i)}}{1+\left[\frac{2\alpha_i(1-\overline{T}_{cr})}{K}\right]^2}, \tag{12}$$

where

$$\psi_i = 1 - \frac{K}{1-\overline{T}_0}\left\{2\frac{\overline{X}_{cr}}{Pe'} + 0.125 + \frac{1-\overline{T}_{cr}}{K} - \frac{1}{\alpha_i^2} - \sum_{n=1}^{\infty}\exp\left[-2\beta_n^2\frac{\overline{X}_{cr}}{Pe'}\right]\frac{\alpha_i^2}{\beta_n^2(\alpha_i^2-\beta_n^2)}\right\}.$$

The values of ψ_i for the first two terms of the series are presented in Fig. 2 as the function $K/(1-\overline{T}_0)$ for different ratios $(T_v-T_{cr})/(T_v-T_0)$.

The average relative underheating in section II of the jet is

$$\vartheta_{cr}^{II} = 2\int_0^1 \vartheta \overline{r} d\overline{r} = 4\sum_{i=1}^{\infty} \exp\left[-2\alpha_i^2 \frac{\overline{X}-\overline{X}_{cr}}{Pe'}\right] \times \frac{\psi_i}{\alpha_i^2\left\{1+\left[\frac{2\alpha_i(1-\overline{T}_{cr})}{K}\right]^2\right\}}. \quad (13)$$

Obviously from this relation when $K \to \infty$ we can obtain as a particular case the Kutateladze formula for a constant jet diameter.

As calculations by (13) show, when $\overline{X}/Pe' > 1 \times 10^{-2}$ we can restrict ourselves to two terms of the series with a sufficient degree of accuracy.

The results of calculating the average relative underheating are presented in Fig. 3a, b, c as a function of $(T_v - T_{cr})/(T_v - T_0)$ for different $K/(1-\overline{T}_{cr})$ and $\overline{X}/Pe'$. As we see from the graphs, the effect of the initial jet temperature on the average underheating is considerable at a critical vapor flow near the jet surface, but the effect of $\overline{T}_0$ diminishes as the section with the critical vapor flow decreases. It is also obvious that when $K/(1-\overline{T}_{cr}) > 100$ the effect of this parameter also practically disappears.

Calculations show that for water vapor even at a vapor temperature of 40°C the value of $K/(1-\overline{T}_{cr})$ is considerably greater than 100. Therefore, for water the average relative underheating is always a function only of velocity and the geometric parameters of the jet. For mercury and potassium vapors the effect of $K/(1-\overline{T}_{cr})$ must be taken into account at a vapor temperature to 500 and 850°K respectively.

When using a jet apparatus as a heater of the working body the relative underheating of the jet is a characteristic of the quality of operation of the apparatus, but with its use for vapor condensation the heat-transfer coefficient is such a characteristic.

From the heat balance on the jet we can obtain after elementary transformations the average heat-transfer coefficient for the section

$$k = -\frac{\lambda+\lambda_t}{2d_0}\cdot\frac{Pe'}{\overline{X}}\ln\vartheta_{av}$$

We will analyze the change of the heat-transfer coefficient upon a change of $\overline{X}/Pe'$ in both sections at constant temperatures of the vapor ($K = \text{const}$) and of the jet at the inlet ($\overline{T}_0 = \text{const}$).

When $\overline{X} \le \overline{X}_{cr}$, expanding the value of $\ln\vartheta_{av}^{I}$ in a series and differentiating, we obtain

$$\frac{dk}{d\left(\frac{\overline{X}}{Pe'}\right)} = \frac{\lambda+\lambda_t}{2d_0}\left[2\left(\frac{K}{1-\overline{T}_0}\right)^2 + \frac{16}{3}\left(\frac{K}{1-\overline{T}_0}\right)^3\frac{\overline{X}}{Pe'} + \cdots\right] > 0.$$

Consequently, in this section the average heat-transfer coefficient increases with an increase of $\overline{X}/Pe'$.

When $\overline{X} > \overline{X}_{cr}$, assuming for simplifying the calculations the average relative underheating to be equal to the first term of the series in Eq. (13), we obtain

$$k \cong -\frac{\lambda+\lambda_t}{2d_0}\cdot\frac{Pe'}{\overline{X}} \times \left[-2\alpha_1^2\frac{\overline{X}-\overline{X}_{cr}}{Pe'} + \ln\frac{4\psi_1}{\alpha_1^2\left\{1+\left[\frac{2\alpha_1(1-\overline{T}_{cr})}{K}\right]^2\right\}}\right].$$

Taking into account that

$$\frac{4\psi_1}{\alpha_1^2\left\{1+\left[\frac{2\alpha_1(1-\overline{T}_{cr})}{K}\right]^2\right\}} \cong \vartheta_{av}\Big|_{\overline{X}=\overline{X}_{cr}} = 1-2\frac{K}{1-\overline{T}_0}\cdot\frac{\overline{X}_{cr}}{Pe'},$$

and representing ln of this value in the form of a series, after differentiation we have

$$\frac{dk}{d\left(\frac{\overline{X}}{\mathrm{Pe}'}\right)} = -\frac{\lambda+\lambda_t}{2d_0}\cdot\left(\frac{\mathrm{Pe}'}{\overline{X}}\right)^2$$

$$\times\left[\sum_{P=1}^{\infty}\frac{1}{P}\left(2\,\frac{K}{1-\overline{T}_0}\,\frac{\overline{X}_{cr}}{\mathrm{Pe}'}\right)^P - 2\,\frac{\overline{X}_{cr}}{\mathrm{Pe}'}\,\alpha_1^2\right].$$

We see from the last relation that when $K/(1-\overline{T}_0) \geq \alpha_1^2$ ($\alpha_1^{max} = 2.4048$) the value $dk/d(\overline{X}/\mathrm{Pe}') < 0$ and consequently an increase of $\overline{X}/\mathrm{Pe}'$ in this section leads to a decrease of the average heat-transfer coefficient for the entire jet.

Thus the analysis permits the conclusion that in the majority of practically possible cases of interest for application the maximum possible heat-transfer coefficient is attained when $\overline{X} \cong \overline{X}_{cr}$ and a further increase of $\overline{X}$ leads to a decrease of the heat-transfer coefficient.

A decrease of the heat-transfer coefficient with an increase of length was obtained in experiments conducted on water vapor by Zinger [3], since they were conducted when $\overline{X} > \overline{X}_{cr}$.

NOTATION

ϑ	is the relative underheating of jet;
k	is the heat transfer coefficient;
j	is the specific vapor flow rate referred to the jet surface;
q	is the specific quantity of heat liberated during condensation of vapor on the jet surface;
T_v and T_{cr}	are the stagnation temperature and critical temperature, °K;
T, T_{sur}, and T_{av}	are the temperature of jet respectively at any point, of the surface, and the mean-integral in cross section, °K;
r_0 and d_0	are the jet radius and diameter;
X	is the coordinate along the jet;
C and γ	are the heat capacity and specific weight of working body of jet;
λ and λ_t	are the thermal conductivity and turbulent thermal conductivity of working body of jet;
$\mathrm{Pe} = (w_0 C\gamma/\lambda)\, d_0$	is the Peclet number;
w_0	is the velocity of jet.

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